Structural Design and Optimisation of the Beijing National Aquatics Centre

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INTRODUCTION

In 2003, as part of Beijing's preparation for the 2008 Beijing Olympic Games, a competition was held to select an architectural design concept and structural solution for an international standard swimming centre. The competition was won by the ”Water Cube Consortium”, lead by CSCEC, with PTW and Arup providing all architectural and engineering services.

The winning architectural concept consisted of a 177m x 177m x 30m regular box–shaped building filled with ”soap bubbles” or ”organic cells”. The structural engineering solution was to support the building by arranging structural members at the boundaries of each ”bubble”, and hence form a complex three–dimensional Vierendeel superstructure.

The creation of the bubble concept was based on a desire for an organic looking building. The structural concept was based on the Wheire–Phelan foam which is currently the best solution to Lord Kelvin's question, posed in 1889, ”How can three–dimensional space be broken up such that the surface area between cells is minimised?” The Wheire–Phelan foam is mathematically formulated and completely repetitive. It is the cutting of the array of Wheire–Phelan foam to make the building surfaces that generates the apparent randomness.

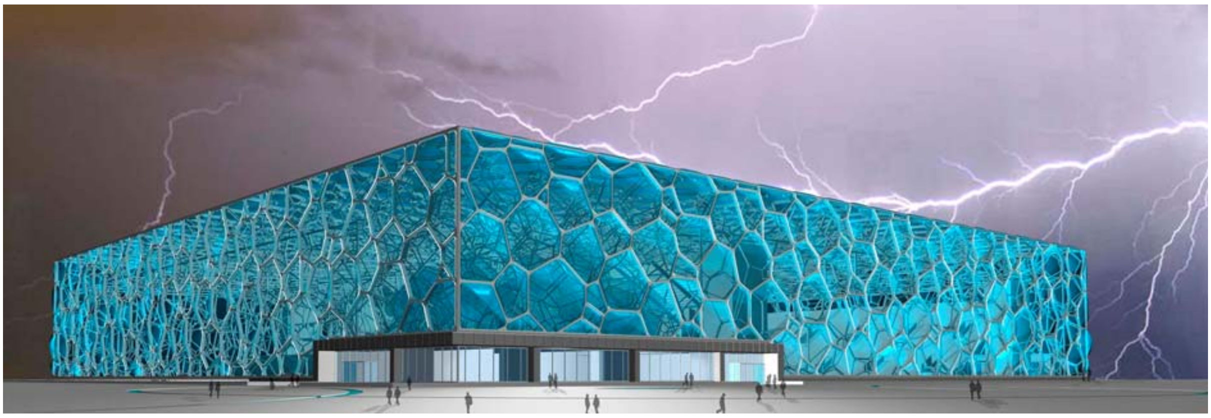

Figure 1 View of Beijing National Aquatics Centre looking towards the main entry.

The structural design of this three–dimensional Vierendeel superstructure was a challenging task. There were 22000 steel members in the superstructure that needed to be designed to resist dead, live, thermal, wind, fire, snow and earthquake loads under many different loading combinations. Computer automation and optimisation techniques were developed to realise the final design solution. The computer automation procedures included structural design, optimisation, and tender drawing creation.

GENERAL DESCRIPTION

The building will be used during the 2008 Beijing Olympic Games for swimming, diving, synchronised swimming and water polo. It will also be used after the Games as a large multi–functional recreation facility for the public. It is situated at the Olympic Green central area on a 6.3Ha site, 305m x 230m in dimensions. The site is located in a seismic zone.

The main pool hall roof spans 140m x 120m with a 60m backspan on one side and a 40m backspan on one other side. The structural depth of the roof is 7.2m and all walls are 3.6m in width. All roof and wall geometry, including both location and widths, were controlled by the fundamental cell size, and a wish to maximise the distance of structural connection nodes from the internal and external façade surfaces. The structural design of this building was governed by strength considerations. For strength design a total of 190 load combinations were considered with only 37 different cross section sizes employed in the entire superstructure.

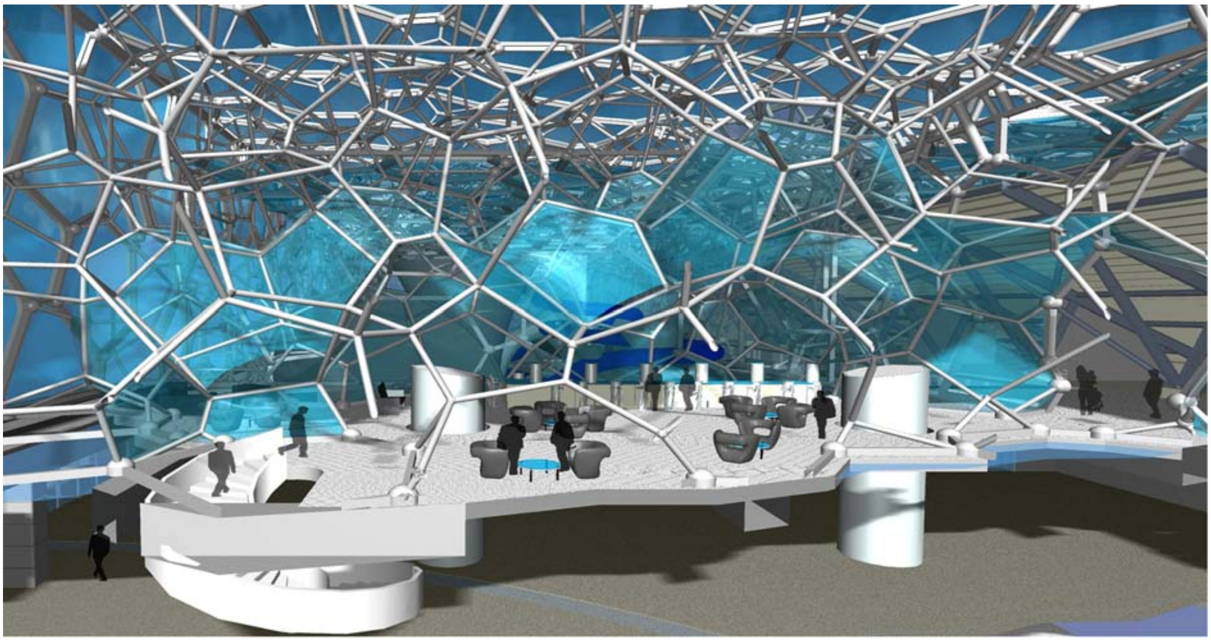

Figure 2 Section through the restaurant showing the three–dimensional Vierendeel structure.

STRUCTURAL LOADING

The roof structure was designed for a uniform live load of 0.3kPa. For robustness seven different extreme out of balance live load patterns were also applied to the roof. Similarly a uniform snow loading of 0.45kPa was considered, together with seven different extreme out of balance snow load patterns. These out of balance snow loads were used in conjunction with wind loads to simulate extreme snow drift conditions for various wind directions. The wind loads were derived from the Chinese wind loading code, given this building is simply a box in shape. Three levels of earthquake were considered during design, with the response spectra being derived from a site–specific hazard assessment. Structure self weight was applied as member loads, together with an additional component at each node location to account for additional connection weight. Superimposed dead load was applied as 0.15kPa for uplift cases or 0.54kPa for downward cases.

LOAD PATHS

The structural system used for the Water Cube is a three–dimensional Vierendeel. As such, changing any one member in the structure causes a redistribution of the forces and moments in the members surrounding it. This also means that the load paths are not statically determinate. The structure predominantly resists load by moment frame action. The main roof span, the internal and external walls, can each be imagined as a four–sided supported flat plate with differing moment fixity along each edge. Each plate consists of thousands of linked cells or bubbles, without triangulation, that transfer load to adjacent cells via bending action. The top and bottom surface of the roof does attract some axial load in a way similar to a truss with a soft web. There is a large amount of redundancy and ductility in this structure which makes it ideal for energy dissipation in this seismic region.

CROSS SECTION SELECTION

The superstructure was grouped into three types of structural element for strength design. The surface edge members were one such group. Their cross section size was architecturally governed as 300 x 300 RHS sections. Only the wall thicknesses of these members changed, with 8 cross section options in total. The internal and external surface members, excluding edge members, form the second group. These vary in cross section from 450 x 300 RHS to 180 x 300 RHS sections, with 13 different cross section options for this group, including geometric and plate thickness changes. The final group of members are the internal ”web” members. This group consisted of 16 CHS cross sections varying from 219mm diameter to 610mm diameter. The maximum plate thickness used anywhere in the design was 40mm while the minimum plate thickness was 4mm. All steel was specified as grade Q345 which is equivalent to grade 350.

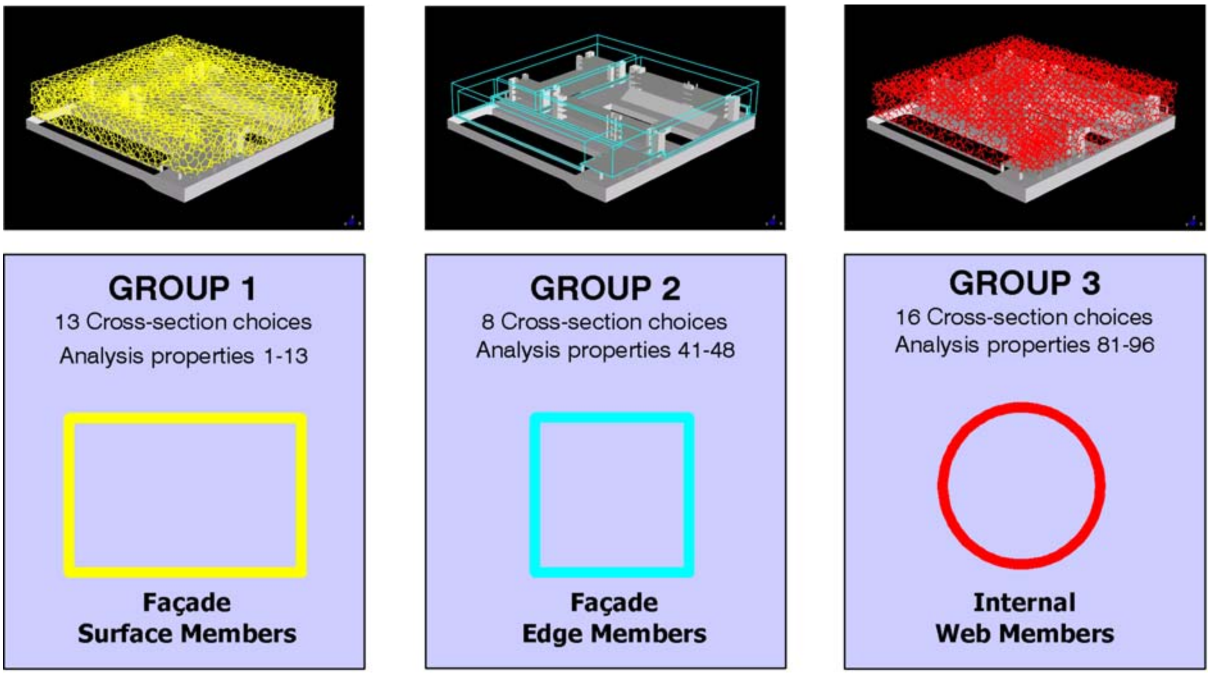

Figure 3 Grouping of structural members for cross section allocation and strength design.

Every member in the superstructure was allocated to one of the three groups above. Each group was then allocated a number of different cross section sizes, specific to that group. Any single structural member in a particular group could potentially be any one of the cross section choices allocated to its group.

DETERMINING CROSS SECTION CHOICES FOR EACH GROUP

For each of the groups, an optimised set of cross section choices needed to be derived, which would be employed in a global optimisation. As the overall design was governed by strength considerations, local cross section strength per kg of steel was maximised for each cross section considered in each group. The cross section choices in a group gradually increased in stiffness from one section to the next, without large jumps between adjacent similar sizes. The gradual stiffness change from one section to the next was crucial for the global optimisation procedure to be effective.

INVESTIGATION OF STIFFENED CROSS SECTIONS VERSUS COMPACT CROSS SECTIONS

During the design process the use of stiffened cross sections to reduce steel tonnage was investigated. The key issue here was the structural performance of stiffened cross sections during the rare event earthquake, which would load the stiffened cross section plastically. One of the most important factors when designing a long span roof is the self–weight behaviour, which can significantly reduce available strength to resist other loads. It was hoped that by using stiffened cross sections, the steel tonnage would be reduced, resulting in reduced member forces and seismic mass, which would in turn lead to further reduced cross sections sizes. This would then lead to further reductions in member forces, and so on, until an optimum lighter solution was achieved. However, it was shown by elastic, inelastic, and plastic finite strip buckling methods that plastically loaded stiffened cross sections buckled before a satisfactory level of ductility could be achieved. As the stiffeners could not be relied upon under plastic loading, the structure had to be designed to resist the rare event earthquake elastically. This lead to an increase in the structural steel tonnage.

Inelastic buckling is buckling up to first yield. It does not track buckling beyond yield. In terms of most design standards, a section that can buckle inelastically at the yield stress would be at least non–compact. However, to classify a section as compact there needs to be a certain amount of stable behaviour beyond yield into the plastic range. For this to be assessed the section needs to be pushed into the plastic range. So inelastic buckling is only valid up to first yield (and can be similar to elastic buckling if the buckling stress is well below yield), while plastic buckling allows us to look beyond first yield into the buckling behaviour of plastically loaded sections.

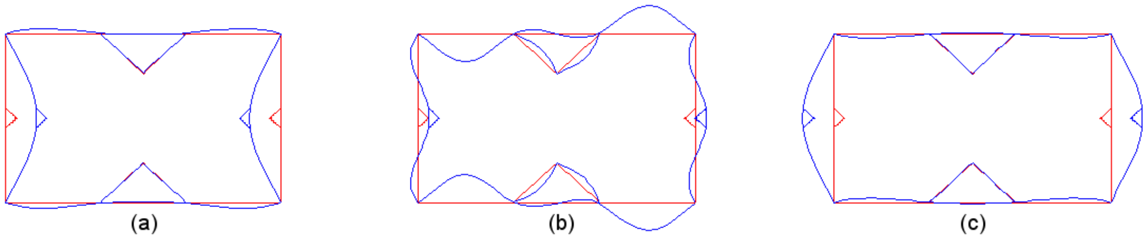

Figure 4 (a) Elastic buckled shape under pure compression; (b) Inelastic buckled shape under pure compression; (c) Plastic buckled shape under pure compression.

The graph in Figure 5 shows the variation of critical buckling stress as the length of cross section buckle is increased out of the page for the cross section shown above in Figure 4. The maximum strain of the cross section at the point of plastic buckling versus buckling length is also plotted on the right hand vertical axis of Figure 5. The elastic critical buckling stress was above the yield stress and so not relevant for design. The inelastic buckling analysis employed an inelastic Winter plate strength curve together with Ramberg–Osgood material parameters to show that the cross section remained fully effective up to the point of Euler buckling; ie, the inelastic critical buckling stress was equal to the yield stress up to the point of Euler buckling. This is equivalent to a non–compact classification in AS4100–1998. The plastic finite strip buckling analysis results show that the plastic buckling strain was equal to the strain at first yield, except for very small buckling lengths and beyond the point at which Euler buckling governs. This confirmed the results of the inelastic buckling analysis, and also showed us that the stiffeners would not provide the necessary ductility to resist the Level 3 seismic event.

The construction of such a sophisticated structure would inevitably result in locked–in stresses and the use of stiffened cross sections would increase fabrication cost and connection complexity. For these reasons, together with the lack of available ductility for stiffened cross section loaded plastically, all cross sections adopted in the final design solution were unstiffened cross sections and they comply with the requirements of a compact section to the Chinese design codes.

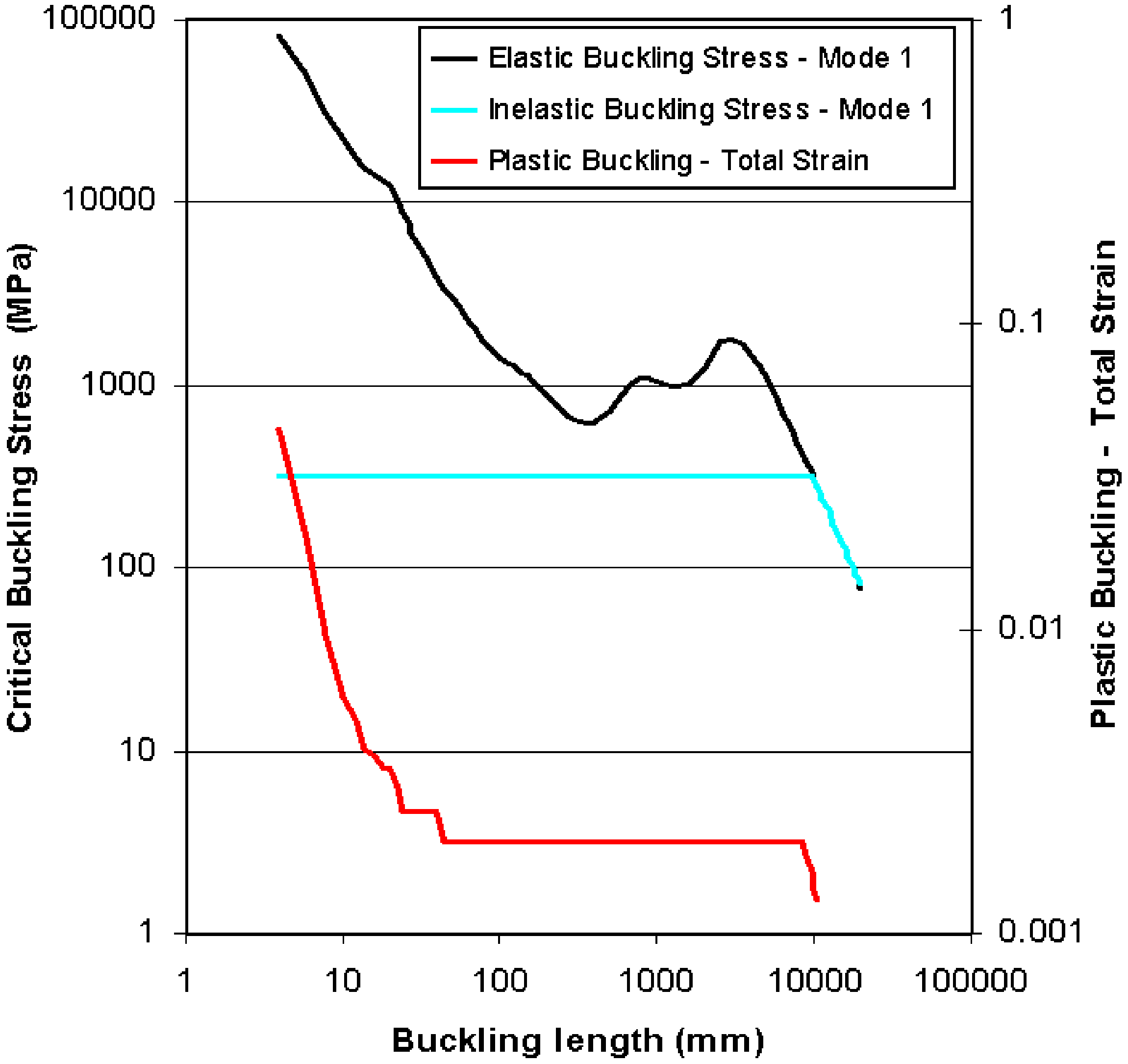

Figure 5 Variation of critical buckling stress and plastic buckling strain with buckling length for elastic, inelastic and plastic finite strip buckling analysis of cross section in Figure 4.

AUTOMATED GLOBAL STRUCTURAL ANALYSIS, DESIGN AND OPTIMISATION OF THE WATERCUBE SUPERSTRUCTURE

The Water Cube superstructure consists of 22000 structural members that each need to satisfy the strength requirements of 11 different Chinese steel design code clauses at 5 points on each member for 190 different load combinations. This equates to 230 million design constraints needing to be satisfied by varying 22000 discrete variables in a strength optimisation to minimise overall steel tonnage. This problem is too large for gradient–based optimisation methods, probabilistic optimisation methods or genetic algorithms and so an alternative method was needed to arrive at a final satisfactory design. The method adopted was a constraint satisfaction method. Strictly speaking this is not optimisation to the purists, however it did result in a significantly lower steel tonnage via an iterative converging process while satisfying all of the design constraints. The automated analysis/design/optimisation process is outlined in Figure 6.

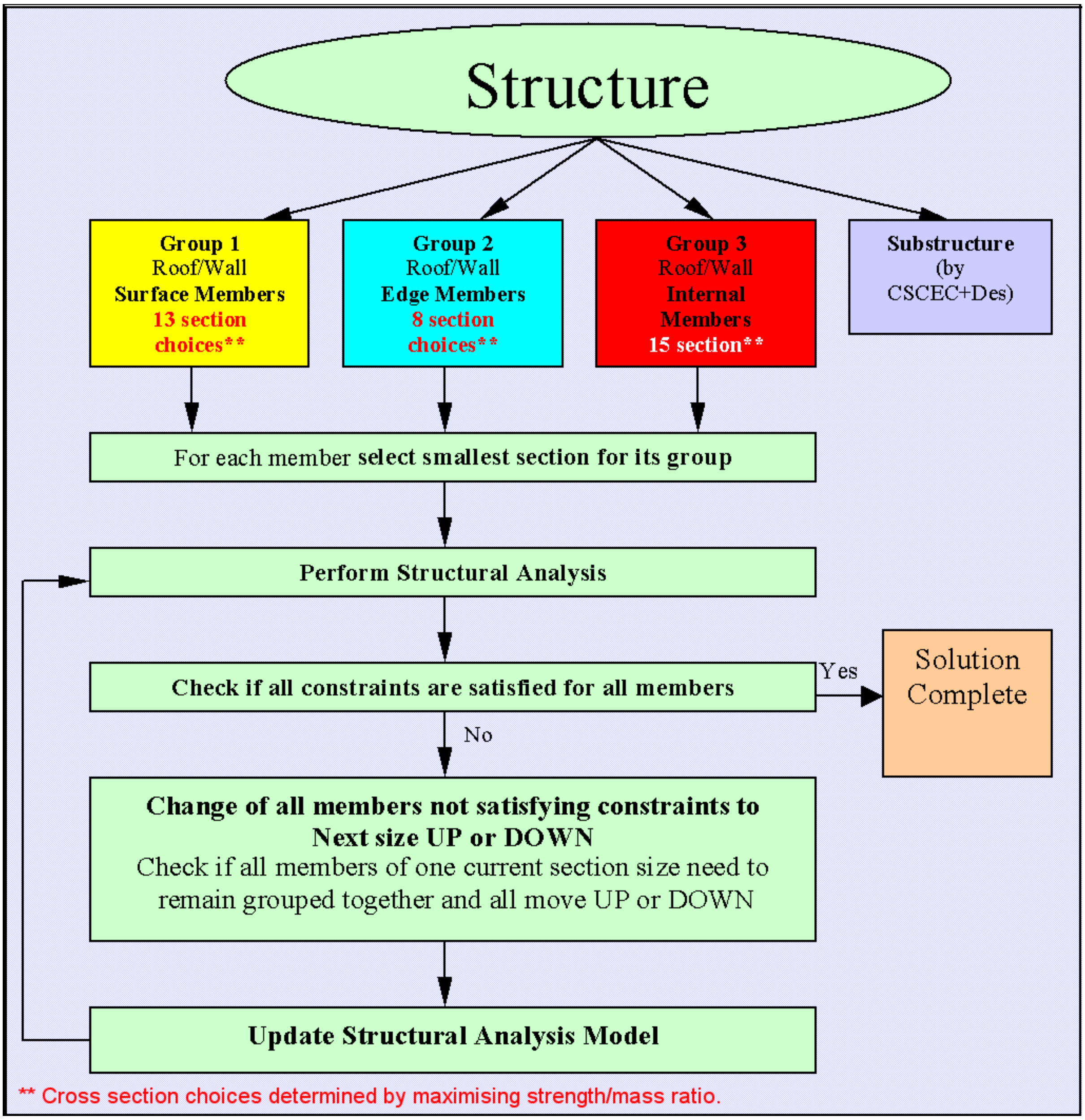

Figure 6 Flow diagram of Analysis/Design/Optimisation procedure.

The optimisation process commenced with each member in the superstructure being set to the minimum strength cross section for its group. A structural analysis was then performed using the Strand7 structural FEA software before the member forces were extracted and strength design checking performed for every member under every load combination. At the conclusion of an iteration, members that were over–utilised were increased in size while members that were under–utilised were reduced in size. Due to the sensitive nature of the structural system, member size increments were limited to one size increment per iteration to control the amount of force redistribution that would occur as a result of stiffness distribution changes. It is for these reasons that the determination of the optimum cross section choices for each of the three member groups was important, as it affects the convergence and result of the global optimisation process. Figure 7 shows the distribution of cross section sizes before and after optimisation.

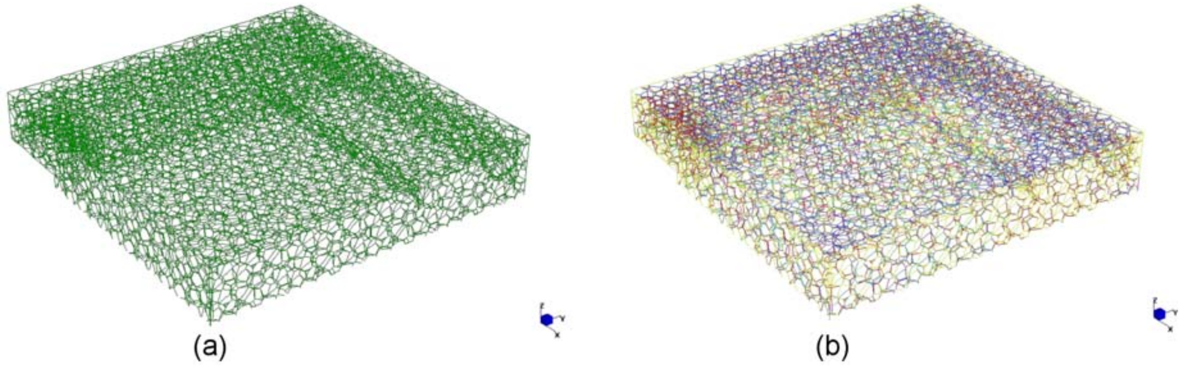

Figure 7 Structure member sizes by colour; (a) before optimisation, (b) after optimisation.

The structural analysis, Chinese design code strength checks, modification of member cross sections after an iteration and creation of a Microsoft Access calculations database described in Figure 6 were all controlled by computer software specifically written for this project using Visual Basic programming. In particular the software developed interfaced directly with Strand7 using its Application Programming Interface (API). This allowed data in the Strand7 analysis model such as the beam element properties to be modified directly, and all analysis solver options to be set and the solver executed directly from the Visual Basic application. Similarly, member forces and moments were extracted directly from the Strand7 results database using API function calls. All strength check results, utilisations, and member stresses were written directly to the MS Access database upon convergence of solution.

There were several key aspects of the design that needed to be built into the automation software. Firstly there was a need to build in upper and lower strength utilisation limits for every cross section choice in all three groups. If a structural member were strength–utilised below its lower strength utilisation limit then it would need to be reduced in size. Likewise if a member were strength–utilised above its upper strength utilisation limit then it would need to be increased in size. A further option developed was the ability for all members of the same cross section size to be moved up or down in size together based on the critical member utilisation for the those members. This allowed different sub–grouping patterns to be investigated to try and rationalise the structure's member sizes into geometrically defined regions. Unfortunately for these investigations no valid solution could be found. Full optimisation of every individual member was required to achieve a valid solution.

The seismic ductility force reduction factors for different seismic cases also needed to be controlled. Built into the automation software was the ability to define independent groups solely for the purpose of allocating different seismic ductility. For final design all seismic ductility groups were set to be the same as the cross section groupings. Each seismic ductility group was allocated an R (seismic ductility force reduction) factor for every load combination by which the seismic forces could be divided to obtain design moments and forces. For non–seismic load combinations R=1. For the Level 1 and 2 earthquakes R was also set to 1. However for the rare event Level 3 earthquake, R=3 was adopted for the compact cross sections. During the optimisation process an equivalent static earthquake load case was applied to expedite the optimisation process for all three earthquake levels. On final convergence a full response spectrum analysis including the substructure was performed in Strand7. Mass participation of at least 90% was achieved in both lateral directions and for the roof itself vertically, with 4424 natural modes included in the solution. The structure was re–assessed for strength based on the results of the response spectrum analysis. A non–linear static pushover analysis was also carried out using Arup's own material and geometric non–linear static dynamic relaxation solver, GSA. Due to the large redundancy in this structure, the use of compact cross sections and the numerous possible plastic hinge formation locations, the structure was shown to comfortably withstand the Level 3 earthquake within reasonable plastic strain limits.

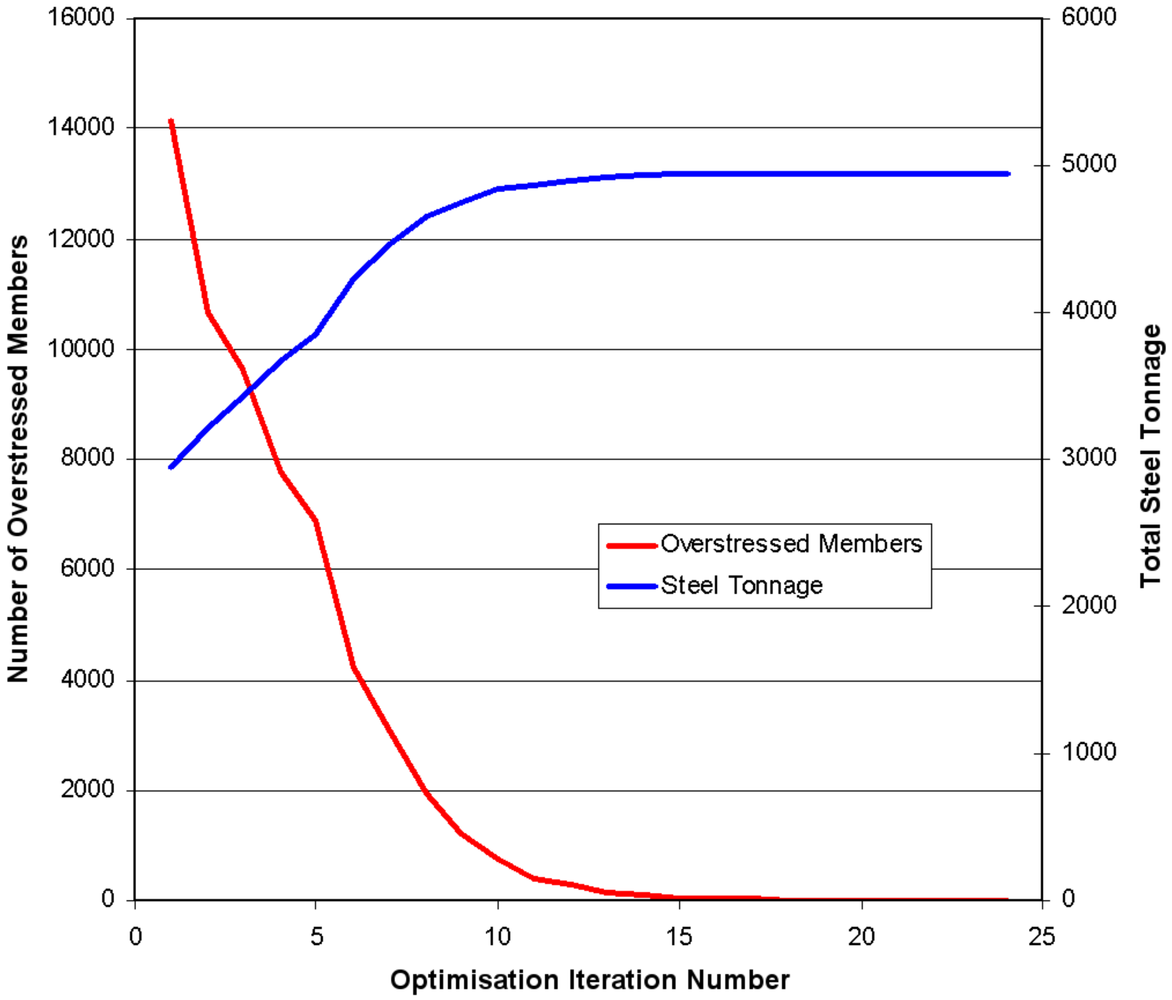

Figure 8 Convergence of global structural optimisation process.

The convergence on a solution occurred rapidly with only 25 iterations required for the final tender design. A complete structural analysis was performed after each iteration due to significant stiffness changes as a result of under–utilised and over–utilised members changing cross section size. Figure 8 shows the convergence of the solution in terms of number of overstressed members per iteration and total steel tonnage per iteration. It was found that different starting assumptions on initial member sizes resulted in different final solutions, with the minimum steel tonnage being achieved by starting at the minimum cross section for all beams. This is not surprising considering how crucial self–weight of structure is in the design of long span roof structures. It was also found that the more continuous the discrete cross section choices for a group were, the more rapidly the solution converged and the lower the steel tonnage. The upper and lower limits of strength utilisation set for each group also had an impact on the final tonnage. It was found that setting both the upper and lower limits as high as possible resulted in the lowest tonnage. Of course if the lower limit is set too high compared to the upper limit, elements could flip–flop between under– and over–utilised, resulting in non–convergence. As locked–in residual stresses from construction will occur, the superstructure was optimised to achieve 80% strength utilisation, leaving 20% utilisation for construction stresses. Figure 9 shows the final strength utilisations contoured on the superstructure while Figure 10 shows the distribution of strength utilisations for the entire structure.

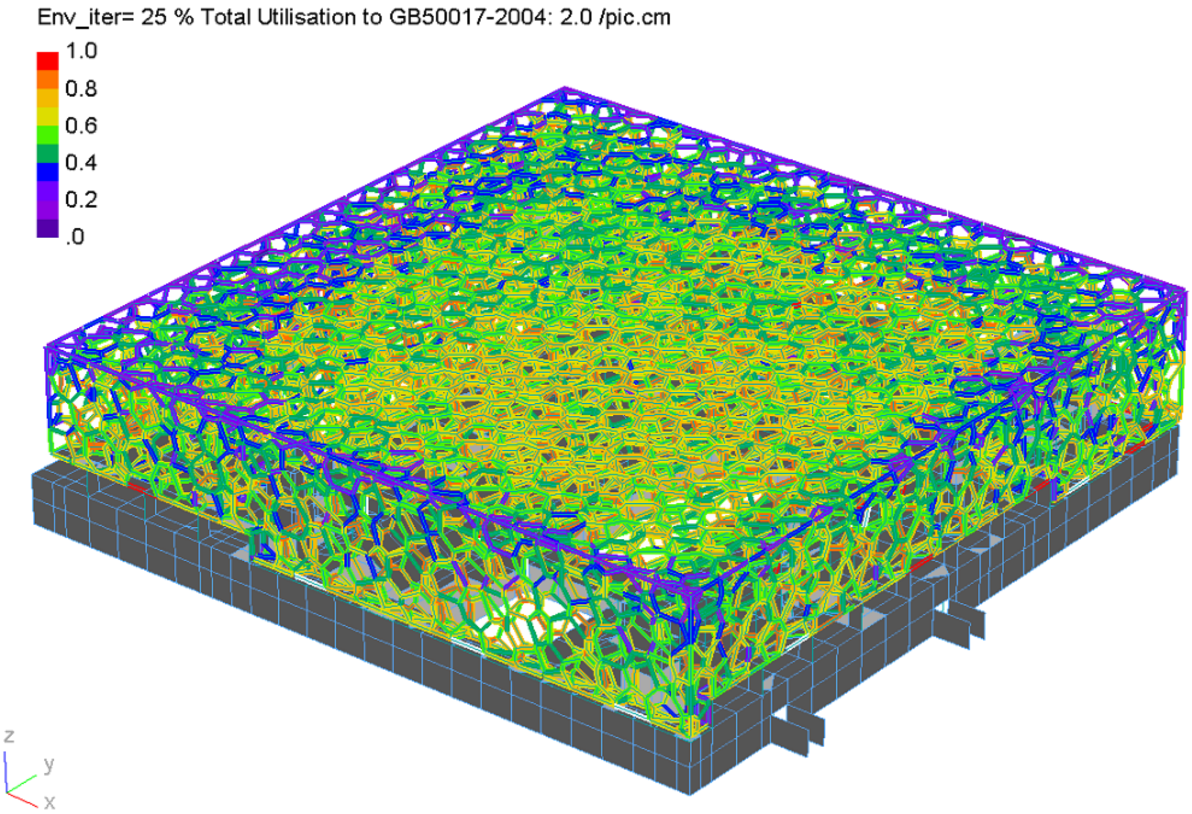

Figure 9 Contour plot of strength utilization for each structural member after optimization.

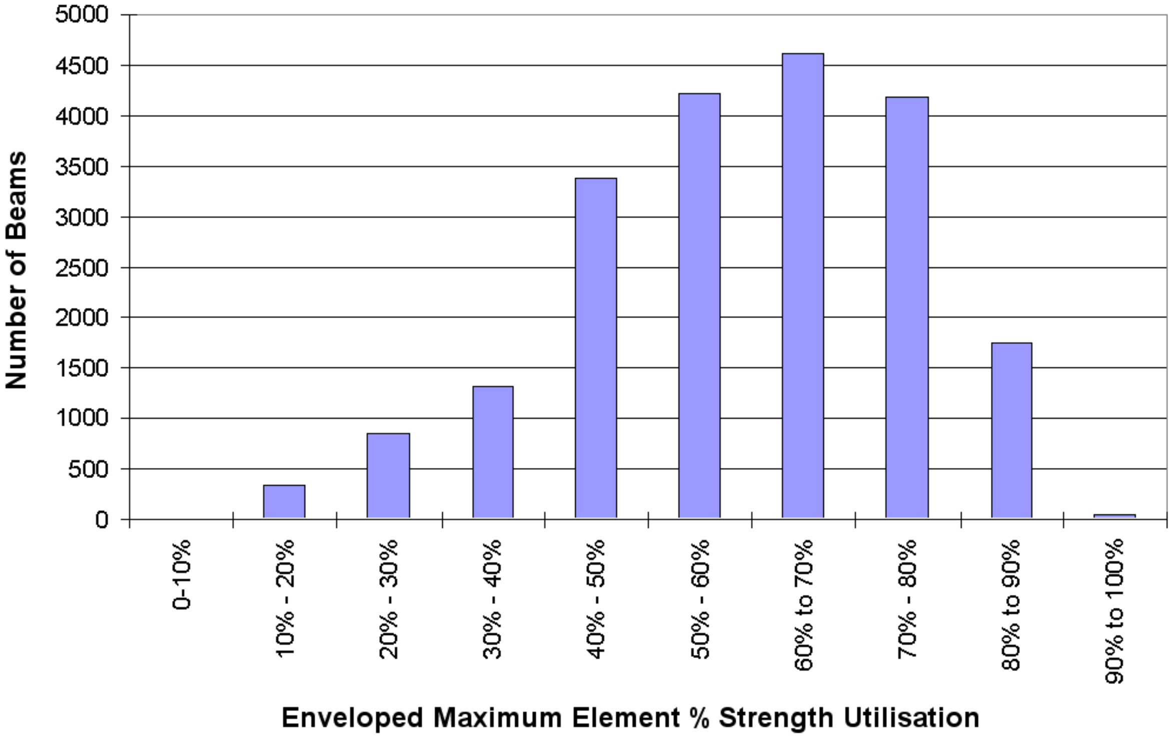

Figure 10 Histogram showing the number of members in each 10% strength utilization band after optimisation.

EARTHQUAKE RESPONSE SPECTRUM ANALYSIS

The structural optimisation and design process assumed a first mode cantilever response to earthquake loads. To apply this simplified loading, a percentage of gravity load was applied laterally with a base shear as calculated by a hand method to the Chinese seismic code for the Level 1, 2, and 3 earthquake events. For the Level 1 and 2 earthquakes the structure was designed to remain elastic with no seismic ductility force reduction used. However, for the rare event Level 3 earthquake a seismic ductility force reduction factor of 3 was employed. The severity of the Level 3 earthquake was one of the main reasons why compact cross sections were used in the final design. The Level 1, 2 and 3 site specific response spectra can be seen in Figure 11. The seismic ductility force reduction factors were incorporated into the automated optimisation process by allowing the creation of subsets of structural members with differing ductility force reduction factors for every load combination.

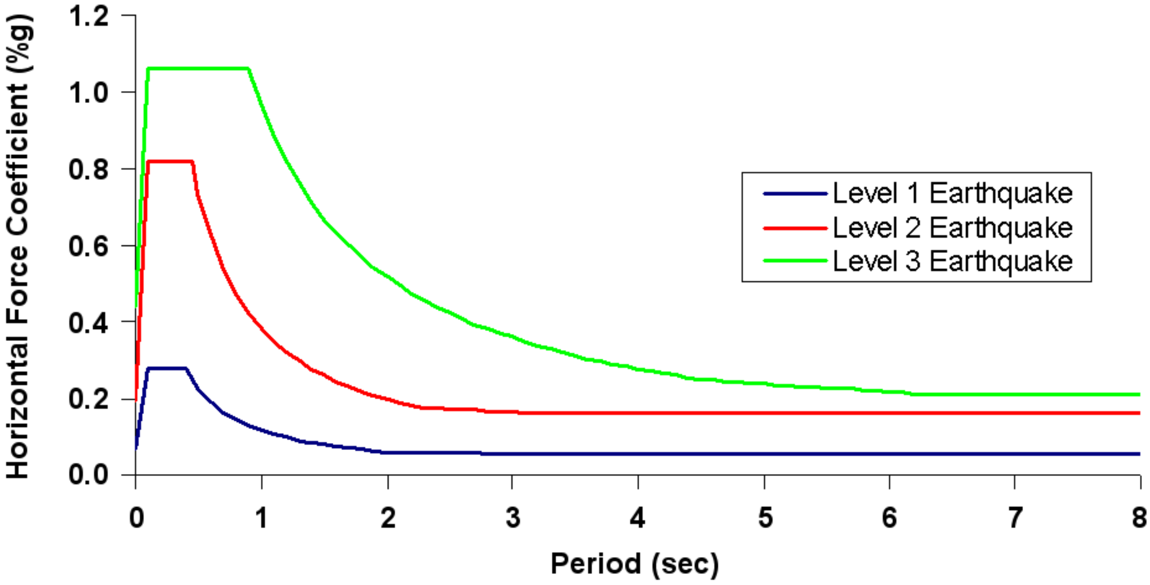

Figure 11 Site–specific Response Spectra for the Water Cube site.

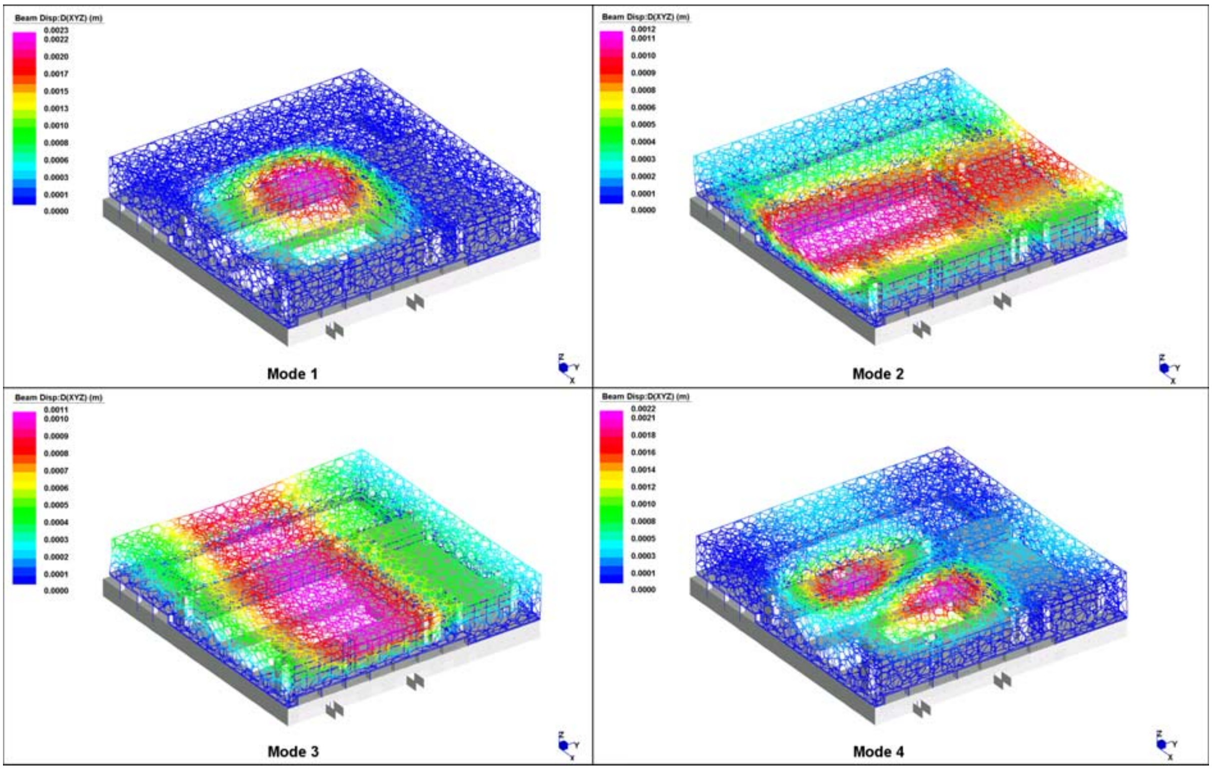

Figure 12 First four natural modes of the optimised structure.

The first four modes of the optimised structure are shown above in Figure 12. Modes 2 and 3 show a similar mode shape to that assumed for earthquake in the optimisation process. However, they only contribute a relatively small amount of mass participation, as seen in Figure 13, while a minimum of 90% mass participation is required by the Chinese seimic design code for response spectrum analysis results to be acceptable. To achieve this 90% mass participation, a total of 4424 natural modes had to be computed. This was a challenge in itself. The solution of a 22000 beam element, 3000 shell element structural analysis model for 4424 natural modes was achieved by solving for small batches of about 30 modes at a time and then overlapping each consecutive batch. As each batch of modes was found, an upward frequency shift was set for the next batch. Only unique modes from each new batch were stored and after each batch the cumulative mass participation was recalculated. Once the 90% mass participation was achieved, the modal solution was complete, and all unique modes from each batch were assembled into a single set of natural frequency mode results that could be employed in a response spectrum analysis. This process was automated by a Visual Basic script specifically written to interface seamlessly with the Strand7 analysis software using its API. An advantage of this method of solution was that computer RAM and hard disk requirements were kept to a minimum. This batch procedure was more efficient than the solution of all the modes in a single run.

The mass participation for each mode together with the cumulative mass participation for all 4424 modes are shown in Figures 13 and 14 respectively. It is clear that this is a special structure in terms of its dynamic properties and modal response. It is worth noting that the vertical (Z) cumulative mass participation appears to be only approximately 60%. This is based on the total structural mass. If just the roof mass is calculated, then the vertical cumulative mass participation for roof modes is in excess of 95% mass participation.

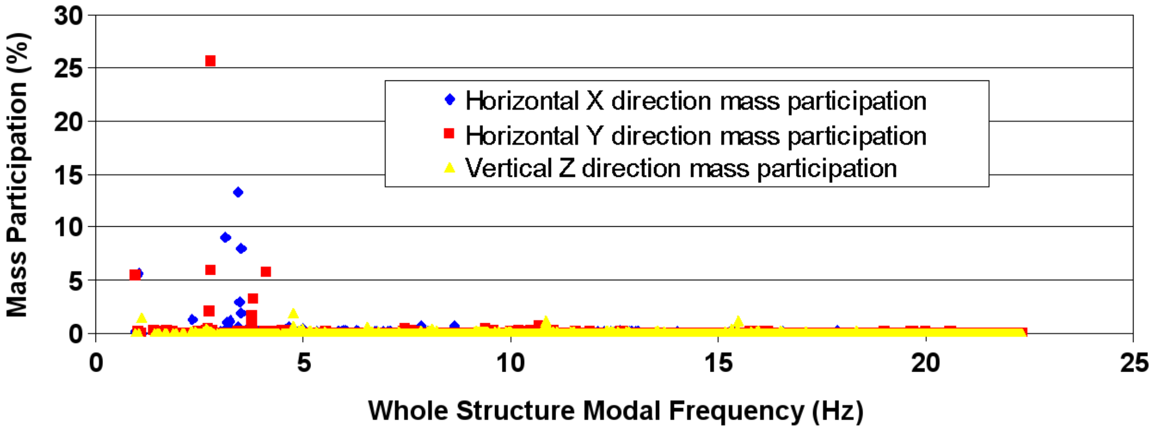

Figure 13 Mass participation of each natural mode of the structure.

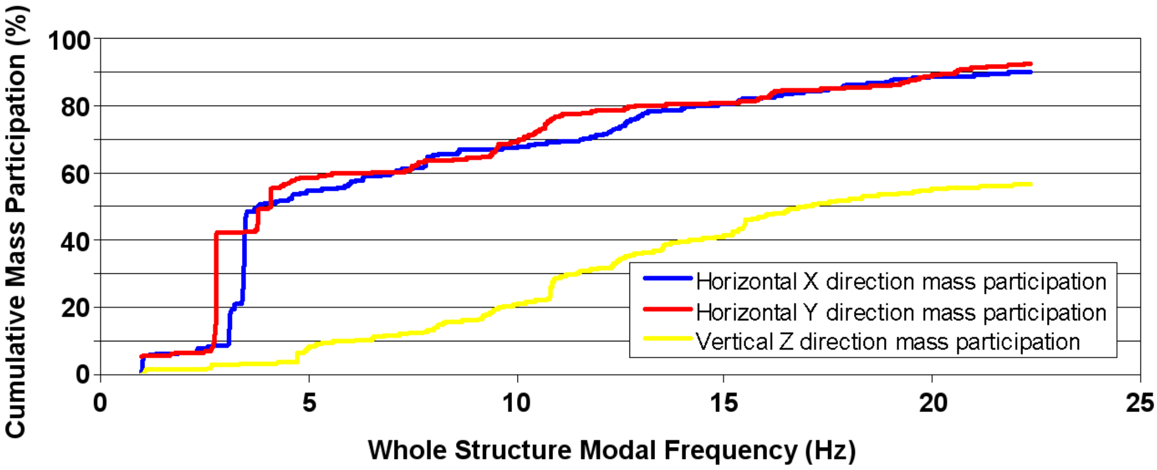

Figure 14 Cumulative mass participation for all natural modes up to 90% mass participation.

A response spectrum analysis was then performed for each of the Level 1, 2, and 3, earthquakes and the final pass strength checks performed on the Water Cube structure using the optimisation software in design check mode, for the more refined earthquake forces.

CONCLUDING REMARKS

The novel three–dimensional Vierendeel structural form, based on the Wheire–Phelan foam, of the Beijing National Aquatics Centre is a world first for building engineering. The structural design was an extremely complex and sophisticated piece of engineering, made possible only by the use of the finest computer and software technologies of today. The automated analysis/design/optimisation computer software developed has since been used on other significant projects in China with great success and will continue to be used on future projects.

CREDITS

1. CAD model images created by Arup+PTW+CSCEC.